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miracle, but the realization of its own character—its own meaning as logical. I do not profess, of course, to have shown that such is the case for *all* propositions; that is a matter which I have not discussed. But in showing the tenability of the hypothesis that practical judgments are of that nature, I have at least ruled out any purely dialectic proof that the *nature* of knowledge as such forbids entertaining the hypothesis that the import—indirect if not direct—of all logical propositions is some difference to be brought about. The road is at least cleared for a more unprejudiced consideration of this hypothesis on its own merits.

JOHN DEWEY.

COLUMBIA UNIVERSITY.

A TOO BRIEF SET OF POSTULATES FOR THE ALGEBRA OF LOGIC

THE algebra of logic as usually developed makes use of an implication relation whose meaning differs somewhat from the usual one. It is commonly referred to as "material implication." Some have supposed that this difference of "p implies q" in symbolic logic from "q can be inferred from p" in Aristotelian logic represents a distinct advance in logical analysis, and that the mere fact of an unusual meaning of "implies" does not necessarily vitiate proof.

For the consideration of any such persons, we here offer a most economical set of primitive propositions, from which all the theorems of symbolic logic, all the theorems of cardinal and ordinal arithmetic, of statics and dynamics, and of various branches of exact science can immediately be derived.

Let p, or q, or r, etc., represent any *true* proposition.

Let $\sim p$ represent the negation of p, "p is false."

(Please note that, although p, q, r, etc., represent *true* propositions, false propositions can equally well be smybolized. If p is some true proposition, $\sim p$ will be some false proposition.)

Let $(p + q)$ represent: "One (but not both) of the two propositions, p and q, is true." Remembering the restriction, we can read $(p + q)$ "Either p is true or q is true."

The fact that p implies q is to be symbolized by $(p \supset q)$. "The idea of implication in the form in which we require it can be defined. The meaning to be given to implication in what follows may at first sight appear somewhat artificial; but although there are other legitimate meanings, the one here adopted is very much more convenient for our purposes than any of its rivals. The essential property that we require of implication is this: 'What is implied by a true proposition is true.' It is in virtue of this property that implication yields proofs. But this property by no means determines whether anything, and if so what, is implied by a false proposition.''[1]

Our definition is as follows:

$$(p \supset q) = (p + \sim q) \quad \text{Df.}$$

''p implies q'' is defined to mean that $(p + \sim q)$. This last expression, according to our use of the symbols $+$ and $\sim$ explained above, represents ''One (but not both) of the two propositions, p and 'q is false,' is true.'' Here p and 'q is false' can not both be true; hence if p is true, q must also be true. Thus implication, so defined, has the requisite property that 'what is implied by a true proposition is true.'

We make one further assumption:

$$p \supset (p + \sim q).$$

This postulate is obviously true. If p is true, then one of the two, p and ''q is false,'' is certainly true. But ''q is false'' is a false proposition, since q is restricted to represent only *true* propositions. Therefore, if p is true, one (but not both) of the two, p and $\sim q$, is true. That is exactly what the postulate states.

No further assumptions are necessary, except the principle stated above, that ''what is implied by a true proposition is true.''

THEOREM 1. $p \supset (p \supset q)$.

Proof: Substitute for $(p + \sim q)$, in the postulate $p \supset (p + \sim q)$, its defined equivalent $(p \supset q)$. Q. E. D.

Theorem 1 may be read: ''Any true proposition, p, implies that p implies any true proposition, q;'' or more briefly, ''Any true proposition implies all true propositions.'' (This symbolic verification of the coherence theory of truth may be noted in passing.)

Theorem 1 is the immediate basis for all further work. Choose *any* true proposition (preferably stated in symbols)—for example, the principle of permutation, $(p + q) \supset (q + p)$.

THEOREM 2. $(p + q) \supset (q + p)$.

Proof: In theorem 1, substitute the posulate, $p \supset (p + \sim q)$, for p, and the theorem to be proved for q. These substitutions give:

$$[p \supset (p + \sim q)] \supset \{[p \supset (p + \sim q)] \supset [(p + q) \supset (q + p)]\}.$$

Since $p \supset (p + \sim q)$ is true, what it implies is true, and we can assert:

$$[p \supset (p + \sim q)] \supset [(p + q) \supset (q + p)].$$

Here again, what precedes the main implication sign is true, and we can assert what it implies—which is the theorem to be proved.

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[1] Whitehead and Russell, ''Principia Mathematica,'' page 98. This passage is quoted from the authors' discussion of *their* definition of implication.

Any other theorem in logic or in mathematics generally can be proved directly from theorem 1 by exactly the same method: substitute the theorem to be proved for q, and the postulate $p \supset (p + \sim q)$ (or any proved theorem) for p.

We submit that this method gives the most economical development of mathematics from a few postulates ever invented. All the true theorems of mathematics can be deduced immediately from one postulate, with the help of a single definition. And *no false theorem* can possibly be proved by this method, because no false proposition can be substituted for q in theorem 1.

It may be objected that the restriction of p, q, r, etc., to true propositions is unusual and objectionable. But by using the sign of negation, any false proposition can also be symbolized. Further, the principal use of mathematical logic is not to discover new theorems of mathematics, but to exhibit the derivation of all mathematics from a few postulates and primitive ideas. The system above outlined is extremely expeditious and convenient for this purpose.

We can assure the reader that in this case the rabbit actually does come out of the hat. Nothing but what we have carefully explained above need be assumed in order to derive all mathematics from one postulate. It is hoped that those who favor new and "somewhat artificial" definitions of implication will ponder this matter with care. The protagonists of "material implication" are especially urged to state their objections to our procedure.

C. I. LEWIS.

UNIVERSITY OF CALIFORNIA.

REVIEWS AND ABSTRACTS OF LITERATURE

La Notion d'Expérience d'après William James. HENRI REVERDIN. Genève et Bale: Georg et Cie. 1913. Pp. xxii + 221.

A Frenchman's analysis generally attains the qualities of clearness and conciseness and sometimes of over-simplification; a fact which, coupled with the very wide influence of William James upon French thought, makes an examination of the great Pragmatist's philosophy by a penetrating writer who enjoyed his personal acquaintance and collaboration doubly interesting and instructive. The enthusiasm of M. Boutroux and the somewhat mystical reverence of M. Flournoy might in point of objectivity occasionally appear to be limitations of the above general dictum. This is not, however, true of M. Reverdin, who, with all his admiration for the "virgin forest" of James, does not hesitate to take measurements of it, even to "pierce well-graveled roads and to trim in the form of a French garden" to quote M. Flournoy's *bête noir.*

In the present writer's judgment it is exactly this penetration of the